



# Visual Models for Fractions

Math & Science Collaborative

November 4, 2013

# Ice Breaker

- Share your name, district and job
- Share a defining moment in your career with regards to math education (either teaching or learning)



# Goals

- Understand how to assist students in developing number sense around fractions
- Gain familiarity with the visual tools, manipulatives and models that help students develop deeper understandings of fractions and operations with fractions



# Norms

- Be Present
- Keep an Open-Mind – Take a Wondering stance
- Participant Productively

# What are Visual Models?

- What are some of the visual models that you have used in your classroom with your students with respect to fractions?
- What are some other visual models dealing with fractions that you have either seen or heard about but may not have used in your classroom, yet?

# Precursors to Fraction Understanding

## Crazy Cakes

- Divide each of these “strange cakes” into two parts with equal areas.
- Is there more than one way to divide the cakes into two parts with equal areas?
- Discuss your ideas about dividing each of the cakes in your small group

# Precursors to Fraction Understanding

## Crazy Cakes

- Read the Case I I in your binder. Phoebe is the teacher in this classroom.
- What are the student ideas about dividing cakes C and D?
- Do the students have any ideas that are different from your ideas?
- How might the student ideas help with dividing cakes H and I?

# Precursors to Fraction Understanding

- Read the unpacked standards for grade 1 and 2 G that are precursors to the fraction standards that begin in earnest in grade 3
- How is the Crazy Cakes Task akin to an advanced version of engaging in these grades 1 and 2 standards?
- So why were the students in the case doing this task in grade 4? Is that appropriate? Why or why not?



# Precursors to Fraction Understanding

- Read pp. 1-9 in *Beyond Pizzas and Pies* for classroom scenario, research and possible classroom activity akin to Crazy Cakes for gr. 2.
- What differences (if any) do you notice when students are asked to shade versus when students are given pre-shaded shapes?

# Precursors to Fraction Understanding

- Examine the top of p. 12 from *Putting Essential Understandings of Fractions Into Practice*
- Based on what you just read in *Beyond Pizzas and Pies*, what do you anticipate as student responses?
- Would you expect any differences in students responses if you altered the prompt on the task to the bottom of p. 12?

# Precursors to Fraction Understanding

- Let's examine Miriam's and Jaden's responses to the two tasks. (pp. 14-17)
- What do you notice?
- What do Miriam and Jaden appear to understand about fair shares for two people?
- How do their responses on the various parts of the task in figure 1.2 conflict with their responses on the parallel parts of the task in figure 1.1?

# Precursors to Fraction Understanding

- Let's examine another student's work.
  - Henri said only a. and b. show brownies with  $\frac{1}{2}$  shaded.
- What information presented so far was familiar to you or similar to your experience with students?
- What information was new or surprising to you?



# Visual Models

- Based on the Crazy Cakes task and the readings and discussions you just completed, what visual models for fractions are used in grades 1 and 2?
- Why do you think those visual models are used and not others?
- Why do you think the fraction standards are so limited in grades 1 and 2?

# Grade 3

- In your binder, please read the standards for **Number and Operations – Fractions** for Grade 3 ONLY at this point.
- Which of these grade 3 standards either specifically reference or allude to visual representations of fractions?
- For the standards that only allude to visual representations, explain how they do so.



# Unit Fractions

- What are unit fractions?
- Why do you imagine this is called a UNIT fraction?
- Why would unit fractions be important first steps into the use of fraction notation?

# Using Probes

- Just what is a probe?
  - A short, highly focused diagnostic assessment
  - Designed to elicit common understandings and misunderstandings of a topic
- A probe has two parts
  - **Selected response:** includes carefully selected choices including the correct response and distractors that will help surface particular misconceptions
  - **Explanation:** Students are asked to justify their response



# Using Probes

- **LEARNER HAT:**
  - Complete the probe for yourself, actually writing out your own explanation for your choice of location.
- **TEACHER HAT:**
  - After you have fully completed the probe, think about it from the student perspective.
  - Think about possible choices students might make on this probe.
  - Think about the explanations the students will give for their choices – both correct explanations and explanations with misconceptions.

# Using Probes

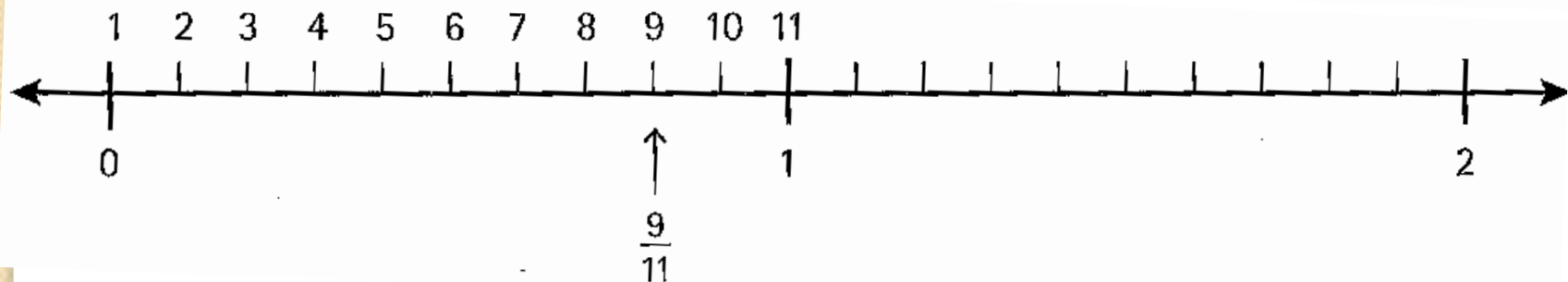
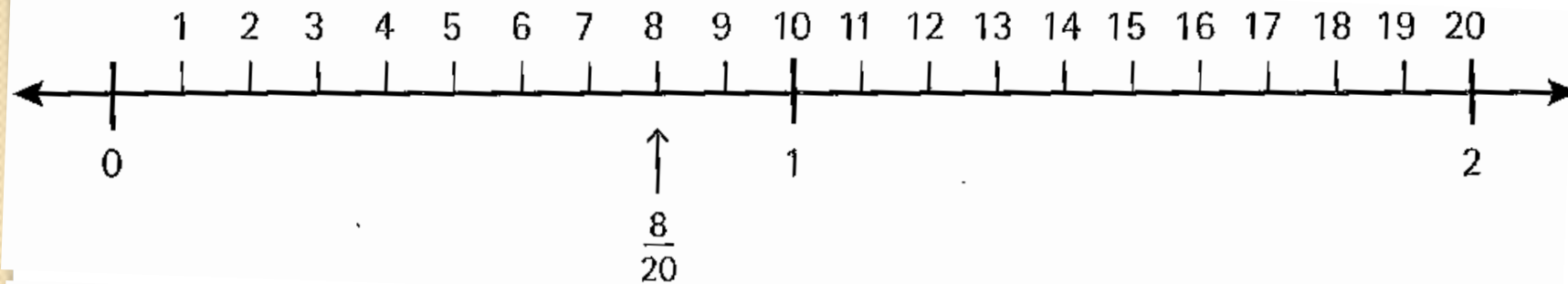
- The selected response part allows for a fairly quick analysis of overall student understanding in the class
- However, the analysis allows for a deeper look at understandings and misconceptions
  - Sometimes, students with the correct selected response, made the choice from incorrect reasoning or by using shallow understandings

# Using Probes

- Examine the samples of student work
- What are the student errors and misconceptions that you see evidence of on this page?
- What are the understandings that you see evidence of?
- Note these next to each sample

# Common Misconceptions

These students were given a number line with tick marks, 0, 1 and 2 marked under it & arrow. What misconceptions might these students have?



# Common Misconceptions

1. Choose the letter  
that shows the  
location of  $\frac{3}{8}$

- ☐ A  
☐ B  
☐ C  
☒ D  
☐ E



Explain why you chose this location:

I choose this place because  $\frac{1}{2}$  would be C  
and  $\frac{3}{8}$  is after half so I choose D.

2. Choose the letter  
that shows the  
location of  $\frac{3}{10}$

- ☐ A  
☐ B  
☒ C  
☐ D  
☐ E



Explain why you chose this location:

Because A is  $\frac{1}{10}$  B is  $\frac{2}{10}$   
C is  $\frac{3}{10}$

# Common Misconceptions

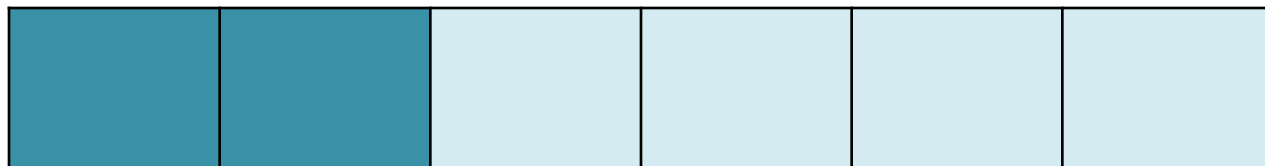
1. Using unconventional notation
2. Redefining the Unit (i.e. ignoring the 0 to 1 unit that is marked)
3. Applying a “two count” strategy
4. Applying a “one count” strategy

# Using Probes

- A probe is NOT ever graded
- It is a diagnostic tool
  - Can be used before a unit to assess prerequisite knowledge
- It can be used for formative assessment
  - Can be used during a unit to see progress towards learning goals and plan next steps
- Can be re-administered near end of unit
  - Do a re-engagement lesson
  - Use a revised version

# Naming the Whole for a Fraction

Mrs. Frances drew a picture on the board.



Then she asked her students what fraction it represents.

- Emily said that the picture represents  $\frac{2}{6}$  . Label the picture to show how Emily's answer can be correct.
- Raj said that the picture represents  $\frac{2}{3}$  . Label the picture to show how Raj's answer can be correct.
- Alejandra said that the picture represents 2 . Label the picture to show how Alejandra's answer can be correct.



# Naming the Whole for a Fraction

- What's the Part, What's the Whole, What's the Fraction?
- Examine *Beyond Pizzas and Pies*
  - Activity 1.2 on pp. 9-13 with Cuisenaire rods.
- And examine *Putting Essential Understandings of Fractions into Practice* pp.31-35
  - How are the activity and the task the same as the activity we just did?

# Progressions Documents

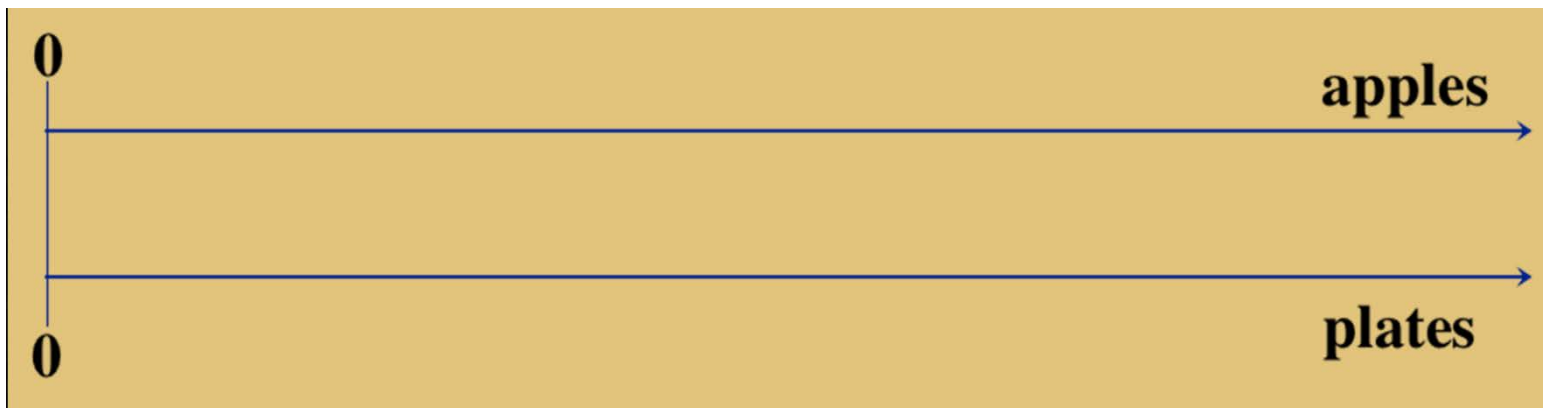
- Progressions for the Common Core State Standards in Mathematics
  - <http://commoncoretools.me/category/progressions/>
- North Carolina Department of Public Instruction: Instructional Support Tools
  - <http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/3rd.pdf>

# Tape Diagrams aka paper strips

- Tape Diagram (CCSS)
  - A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
  - See page 4 of the Progression document

# Double number lines

- Double Number Line
  - Parallel number lines, drawn so there is one for each quantity in question (usually two but can be more than two). The number lines are “hinged” at 0.
  - For our purposes in the fractions seminar, these number lines will also have the same whole.



# Comparing Fractions and Finding Equivalences

- Game : Capture Fractions **variation**
  - Lay all the fraction cards in your deck out on a flat surface. There are 40 cards in the deck.
  - Cards should be face UP, in some sort of rectangular array.
  - Choose two cards to compare to one another.
    - One fraction value can be larger than the other, or the values can be the same.
  - Prove how the fractions relate
  - Create a diagram on your tracking sheet and provide mathematical explanation to your partner to

# Comparing Fractions and Finding Equivalences

- Game : Capture Fractions **variation**
  - You and your partner can use any of the tools and / or visual models that we have discussed or used today
    - Limit yourself only to those tools in order to *increase your familiarity with them*
  - You may assume the fractions are referencing the same whole, a whole of your own choosing!
  - In the course of the game, you must deal with equal as well as greater than / less than

# Debrief on the game

- Does the game support the student's acquisition of the standard 3.NF.3?
- What parts of the standard does the game support?
- How does the game support the acquisition of the standard?

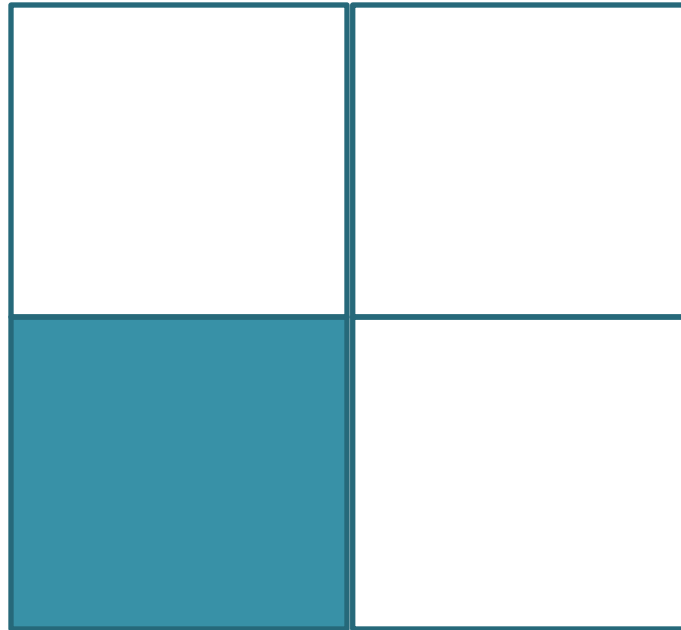
# Debrief on the game

- What can the facilitator do to support the student's while they are playing this game?
- Discuss what the teacher does or does not do while students are playing the game
- How can the teacher actions related to the game support a student-centered classroom climate and the norms and/or routines?



# Comparing Fractions with a Different Whole

Bryce said, “This shows that  $\frac{1}{4}$  is greater than  $\frac{1}{2}$ .”





# Comparing Fractions with a Different Whole


- Illustrate what Bryce's drawing might look like if he had used:
  - Rectangles or Circles or another area model
  - Pattern blocks
  - Tape diagrams / fraction strips
  - Number line or Double number line
  - Cuisenaire rods
- Use another one of these tools to back up your choice(s) for part b

# Fraction Equivalence

- Fair share contexts are a natural way to get students to consider fraction equivalence

*12 people share 4 cakes at a holiday party*

- How will children think about sharing these cakes?

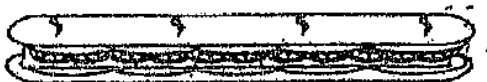
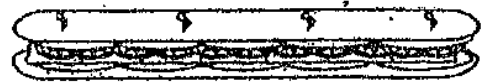
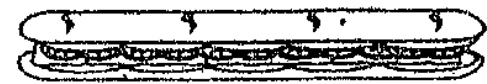


“Many teachers seem to believe that fraction answers are incorrect if not in simplest or lowest terms. This is unfortunate.” (Van de Walle)

# Standards for Mathematical Practice

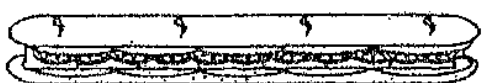
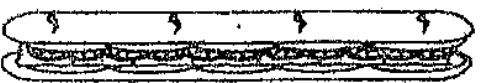
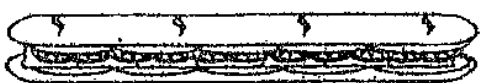
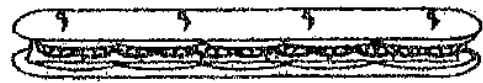
- Read
  - Elementary Elaborations of Standards for Math Practice 1, 3, 4, and 5 only.
  - Highlight the key ideas from your perspective
- Turn and Talk
  - About what you highlighted as key ideas, then
- Respond to the following question
  - To what extent do you think MP 3 is embedded in the daily work of teachers and students?

# Submarine Sandwiches



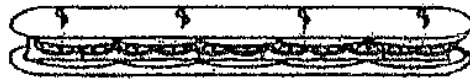
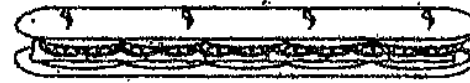
4 kids

Museum of Natural History



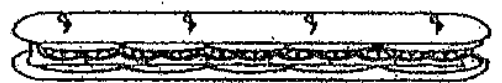
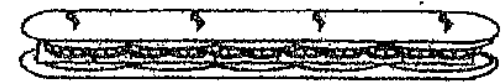
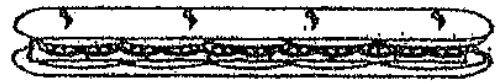
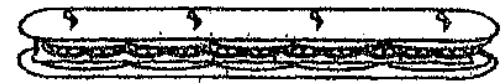
8 kids

Ellis Island and the Statue of Liberty



5 kids

Museum of Modern Art



5 kids

Planetarium

# Submarine Sandwiches

## Gallery walk

- View each poster for a minute or so with post-it notes in hand.
- Provide questions or constructive feedback on the posters you see
- Put the sticky right on the poster.
- Read the feedback on your own poster.

# Submarine Sandwiches

- Congress
  - Our work
  - Student work
- Debrief about content standards
  - Which of the grade 3 content standards did see evidence of throughout the task, the gallery walk and the congress?
- Debrief about practice standards
  - Of the four SMPs that we are focusing on, which of these did you feel like you engaged in **to a great extent?**



# Submarine Sandwiches

- Video of students in the Congress
- Focus on one Content Standard combined with one SMP and plan how to implement one idea or variation of it from today in your class

# Homework I

- Read chapter 2 and 3 in *Beyond Pizzas and Pies*
- See next slide...

# Homework 2

- Read
  - The Standards and the Elementary Elaborations of Standards for Math Practice 1, 3, 4, and 5 only.
  - Highlight the key ideas from your perspective
- Respond to the following questions (on wiki or in written form)
  - To what extent do you think MP 3 is embedded in the daily work of teachers and students?
  - Which of these four Mathematical Practices (1, 3, 4, 5) do you feel like you engaged in during our first day together? Justify your choice(s)?

# Homework 3

Complete the following math task, making sure to use at least one visual representation.

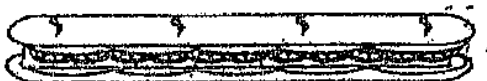
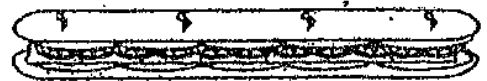
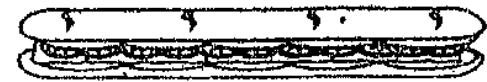
*My class from last year traveled on a field trip. There were subs for each group. (See next slide for illustration)*

- *The first group had 4 kids and 3 subs to share equally*
- *Second group had 5 people and 4 subs*
- *Third group had 8 people and 7 subs*
- *Last group had 5 people and 3 subs*

*When they returned from the field trip the students began to argue that the distribution of subs had not been fair, that some children got more to eat than others. Were they right or did everyone get the same amount? If they were right, who got the most to eat and who got the least? Justify your responses.*

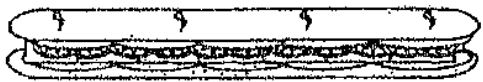
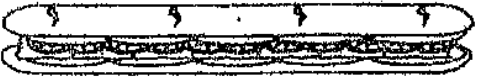
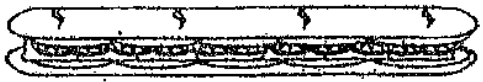
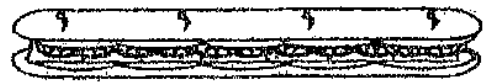
*Make a poster of your work and response for a gallery walk that will take place in AM of Day 2.*

# Submarine Sandwiches



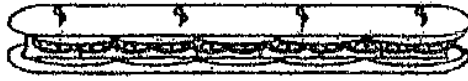
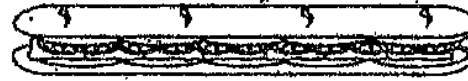
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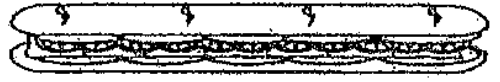
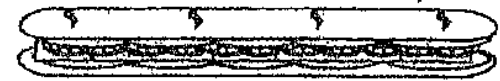
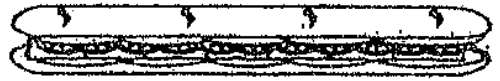
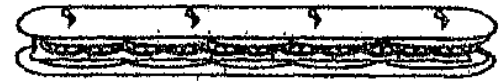
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